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Simplified M-E Approach for the Design of Flexible Pavement Structures

تصميم مبسط للرصفات الإسفلتية باستخدام التوجهات التجريبية والنظرية

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Abstract

This paper presents a simplified design procedure, which depends mainly on finite elements analysis considering the same input variables used in the traditional AASHTO empirical design methodology. The multiple regression equation developed considered the subgrade resilient modulus (Mr), Equivalent Single Axle Load (ESAL) repetitions, and the dynamic behavior of flexible pavements by incorporating vehicle speed. The outputs of this model are given in terms of structural number (SN) making it easier for the designer to deal with the various pavement thickness as per the traditional AASHTO procedure. Based on the model developed for static loading, SN values were approximately -6 to 7 percent compared to those of AASHTO design methodology depending on number of 18 kips single axle load repetitions. In the analysis of the dynamic behavior of pavements, SN dropped up to 30 percent for the practical design speeds that could be considered for pavement design purposes.

Key words: Flexible pavement, design, finite elements, mechanistic, empirical, dynamic analysis.

ملخص

تمثل هذه الورقة العلمية تصميم مبسط للرصفات الإسلفتية باستخدام العناصر المحددة بالإضافة إلى المتغيرات المستخدمة بالطريقة التقليدية لتصميم الطرق حسب مواصفات الجمعية الأمريكية للطرق والمواصلات AASHTO. المعادلة التي تم استنتاجها لهذا الغرض متعددة المتغيرات، وتشمل إستخدام معامل القوة للتربة الطبيعية (Mr) ومعاملات تحويل المحاور للمحور التمثيلي، وكذلك سرعة المركبات على الطريق المقترح للتصميم. وقد أعطيت مستخرجات المعادلة بدلالة المعامل الإنشائي SN للتسهيل على المصمم لايجاد سماكات الرصفة بالطريقة التقليدية المتعارف عليها. يتبين من نتائج هذا البحث لحالة التحليل الاستاتيكي أن النتائج كانت بحدود ٦- إلى ٧% من تلك النتائج المستخدمة بإتباع آلية التصميم الخاصة ب AASHTO وذلك معتمداً على عدد المحاور الفردية من نوع 18 Kips. أما بالنسبة للتحليل الديناميكي، فقد أظهرت النتائج أنها أقل بمقدار ٣٠%، وذلك إعتماداً على مدى السرعة المتوسط لأنواع الطرق المختلفة.

1. Introduction

The recent fast growth of the number and type of vehicles created unpredictable problems in flexible pavement structures. The need to explore the effectiveness of the traditional design approaches were questioned specially those methods which depends on empirical analysis of observed loading and distress data to estimate damage. Flexible pavement performance is influenced by factors such as material properties, the environment, traffic loading, and construction practices. Pavement design depends heavily on empirical relationships based on experience and field tests such as the American Association of State Highway Officials (AASHO) Road Test. Relationships developed between traffic loading and pavement performance from this test are recognized to apply only to the conditions under which they were developed. Damage conditions created by new vehicle characteristics and configuration will be different than those used in the AASHO Road Test. Many attempts were done to derive or adjust the current load equivalency factors.

Analyses of multilayered pavement system were utilized as a part of incorporating pavement mechanistic response using parameters such as stress, strain, or deflection to estimate pavement damage. Analysis considered vertical strain on top of subgrade, the tensile strain at the

bottom of the pavement layer, the surface vertical deflection, and the tensile stress in a concrete pavement. Several analytical models were developed for the solution of multilayered pavement system based on either elastic layer theory or finite elements. Calibrations of those analytical methods are based on the comparison with measured response parameters obtained from field tests.

2. Research Significance

This research is intended to develop a simplified approach for the design of flexible pavement structures based on mechanistic analysis using finite element technique and the common empirical parameters used by AASHTO Guide for Design of Pavement Structures. This technique will enable highway engineers to design a pavement structure with consideration that fits the changing parameters in the field; at the same time, it takes into consideration the typical input parameters considered in the AASHTO empirical design methods.

3. Literature Review

A study was conducted on the investigation of the effect of several parameters such as axle type and load, tire pressure, and speed on the asphalt concrete pavements (Hudson et al. 1992). In this study, single dual tire axle, tandem axle, and tridem axles were used. Three axle load types and two inflation pressures for each were used. In addition, two speeds were used, namely 8 km/hr and 72 km/hr (5 and 45 mph respectively). The research concluded that the axle load causes the most damaging effect with small additional damage caused from increase in tire inflation pressure and increase in speed.

A field study was conducted to study the effect of vehicle speed and axle load using a single unit truck with three axles (Papagiannakis et al. 1992). Speeds considered in the study were 20, 40, and 50 km/hr (12.5, 25, and 31.3 mph, respectively). Three axle loads were also used. This study indicated that the measured tensile strains at the bottom of the Asphalt Concrete (AC) layer exhibited a large sensitivity to vehicle speed. The measured strain at a vehicle speed of 50 km/hr was roughly

tripled at a reduced vehicle speed of 20 km/hr. Consequently, the calculated load equivalency factors were found to decrease with increasing vehicle speed as a clear consequence of the time-dependant behavior of the asphalt concrete. It should be mentioned that this conclusion is not consistent with that of Hudson et al. (1992) with regard to the effect of speed on the pavement structure.

The new mechanistic-empirical design approaches include both mechanical modeling and performance observation in determining the required pavement thickness for a set of design conditions. Mechanical models are based on elementary physics and determination of pavement stresses, strains, and deflections as a result of wheel loads. The use of empirical methods has been established and widely recognized approach to thickness design; however, their disadvantages are becoming more prevalent with the increase in understanding of mechanical-empirical design (Timm et al. 1998).

The revisions of the new 2002 AASHTO 'Guide for Design of Pavement Structures' are based on mechanical-empirical procedures. There are several advantages of mechanical-empirical procedures over the traditional empirical procedures which mainly enable (Timm et al. 1998, NCHRP 2002):

- Consideration of changing load types (new truck axle arrangement and loads),
- Better utilization and characterization of available materials,
- Improved performance predictions,
- Better definition of the role of construction by identifying the parameters that are most influential over pavement performance,
- Relation of material properties to actual pavement performance,
- Better definition of existing pavement layer properties, and
- Accommodation of environmental and aging effects of materials.

Dere et al. (2005) studied the failure prediction of skewed jointed plain concrete pavements using three dinemtional finite elements analysis. This research was performed to determine if the frailure of skewed joined plain concrete pavements could have been predicted using a 3D finite element analysis. The finite element analysis predicted not only the failure of the pavement, but also the correct orientation of the crack.

Fang et al. (2004) examined the characterization of flexible pavement rutting using creep model-based finite element analysis. A matrix of Finite Element Analysis (FEA) was carefully designed so that pavements with low and high traffic volumes, low and high rutting levels, and different structural layer thickness combinations were all included. Transverse wheel wander distribution and non-uniform tire contact stresses were used in modeling traffic loads. The effects of pavement shoulder conditions and high tire pressure were also examined. New criteria were developed for identifying pavement layer failures based on evaluation of transverse surface profiles. Good agreement was found between failures predicted by the criteria and field observations on 20 in-service pavements. Software tools were developed to facilitate the use of the new failure criteria.

Hadi et al. (2003) studied the non-linear finite element analysis of flexible pavements. This research study was being undertaken to incorporate the realistic material properties of the pavement layers and the moving traffic load in the analysis of flexible pavements using the finite element theory. As a preliminary step taken in this direction, a pavement structure, where field measurements have been carried out when subjected to a cyclic loading, was selected and modeled as a finite element model. The analysis was carried out using the finite element computer package ABAQUS/STANDARD, when this pavement model was subjected to static and cyclic loading while considering the linear and non-linear material properties of the pavement layers. The results indicated that displacements under cyclic loading when non-linear materials were present, were the closest to field measured deflections.

Brill et al. (2003) analyzed the Federal Aviation Adminstration (FAA) thickness design procedures for modeling rigid overlays using three-dimensional finite element method. The paper discussed the modeling of rigid overlays as a replacement for the layered elastic models used in FAA thickness design procedures. The mesh with incompatible modes elements was tested against known solutions to demonstrate its basic accuracy for multi-layered problems, in particular, slab on slab overlays with sliding and partially bonded interfaces. The work showed how a 3-D-FE model could be applied successfully to airport pavement structure, even for relatively complex problems such as rigid-on-rigid overlays.

Bhutta et al. (1998) analyzed the M-E design procedure for geosynthetically stabilized flexible pavements considering a secondary road pavement section build as a part of realignment of Route 616 and 757 in Bedford County, Virginia to evaluate the performance of geosynthetically stabilized flexible pavements. The measured pressure at the base course-subgrade interface for the geotextile-stabilized sections was lower than the geogrid-stabilized and control sections, within a specific base course thickness group. This finding agreed with other measurements, such as rut depth, ground penetration radar survey, and falling weight deflectmeter survey.

Novak et al. (2003) compared the near-surface stress states in flexible pavements using measured radial tire contact stresses and ADINA software. The finite element code ADINA was used to identify the three-dimensional stress states in a typical flexible pavement configuration, resulting from measured radial tire contact stresses. The predictions showed that measured radial tire contact stresses resulted in stress states being both larger in magnitude and more focused near the surface than those obtained from traditional uniform vertical loading conditions. In terms of effects of possible pavement damage mechanisms, predicted high near-surface shear stresses may be a part of an explanation for near-surface rutting failure modes, as supported by near-surface slip planes seen in the field.

Sukumaran et al. (2002) tested the suitability of using california bearing ratio test to predict resilient modulus. The paper documented the current state of the knowledge on the suitability of this empirical approach. In addition, the paper documented the use of finite element analysis techniques to determine the California Bearing Ratio (CBR). The stress-strain response of the various soils was simulated using an elasto-plastic model. The constitutive model employed was the classical Von-Mises strength criteria with linear elasticity assumed within the yield/strength surface. The finite element techniques employed were verified against available field and laboratory test data. The model was then utilized to predict the CBR of various soils. The empirical relationship between CBR and resilient modulus would then be investigated based on the results obtained from the three dimensional finite element analysis, and its suitability for flexible pavement design would be evaluated.

Generally, mechanical-empirical designs rely primarily on the mechanics of materials. The simplified design procedure proposed depends mainly on finite element analysis with input module depending on characteristics of material properties as of AASHTO methodology of the 1993 code (AASHTO 1993). The structural Analysis program SAAP 2000 was used as a tool to build and analyze the pavement layers. Results were calibrated using the AASHTO Guide for Design of Pavement Structures, 1993.

Previous research considered several approaches in model formulation and assumptions (Hewany et al. 1998, Aliet et al. 1998, Roberts 1989, Ioannides and Khazanorich 1998, Mamlouk et al. 2000, Tielking and Roberts 1989, Das and Pandey 1999, Hudson et al. 1998, Sven et al. 1992, Bright and Mays 1996). The dynamic vehicle-pavement-foundation interaction effect is significant for the analysis of dynamic response of pavements subjected to moving loads (Siddharthan et al. 1998, Wu and Shen 1996, Zafir et al. 1994, Lin and Gazis 1994).

Early methods based on theoretical approach were dealt with a homogeneous half-space having an infinite large area and depth with the top representing time imprint. Development of multilayer theory by

Burmister started with two layers and later applied to multilayered with the advancement of computers. Other techniques were later developed such as the finite element technique. The Asphalt Institute was one of the first to adopt a multilayer elastic system; wheel loads were assumed to be applied through the tire as a uniform vertical pressure and spread by the different components of the pavement structure and eventually the subgrade. Stresses and strains are evaluated and analyzed in the asphaltic layer based on the criterion for maximum tensile strain at the bottom of the asphalt layer and the maximum vertical compressive strains at the top of the subgrade; thicknesses are developed for the different layers.

A computer program was developed to solve stresses, strains, and displacements as well as the development of elastic layer constants utilizing multilayer elastic analysis (Lin and Gazis 1999). Computations derived from the elastic theory were used for the estimation of vertical subgrade strain. This relates the allowable vertical subgrade strain value for a given number of 18-kips axle load repetitions. The same terminology was used for development of limiting tensile strain criteria for fatigue cracking.

A three different homogenous linear elastic models, mainly multilayered pavement system (Burmister layer), Winker spring bed, and Kirchoff medium-thick plates were analyzed (Mamlouk et al. 2000). Pavement members were described using nonlinear, viscoplasticity concepts. The paper suggested that an equivalent plate with parameters as defined by Ioannides could replace two plates resting on one another. In addition, "n" spring beds in series can also be replaced by one equivalent spring bed with effective spring stiffness. A program was developed allowing the user to model a layer using different models.

Tielking et al. (1989) studied the analysis and design optimization of flexible pavement. A project-level optimization approach was developed to minimize total pavement cost within analysis period. This approach enables the designer to select the optimum initial pavement thickness, overlay thickness, and overlay timing. The developed model combined the AASHTO design procedure and the mechanics of multilayer elastic solution.

Since the seventies, many mechanistic-empirical models have been developed based on elastic and viscoelastic theories and finite element analysis technique. Bisar, Chevron, Elsym and Kenlayer are examples of computer programs incorporating these types of models.

4. Model Formation

The three dimensional analysis method is used for developing the proposed model. SAP 2000 software was used as a finite element tool for building and analyzing the model. The model proposed in this research is composed of three layers, asphaltic concrete layer, base layer representing base and subbase, and subgrade layer. The dimensions of the pavement section considered are 7.5 m (25 ft) along the length of the highway and 7.5 m (25 ft) in the transverse direction, representing a typical two-lane highway (approximately 3.75 m each). Figure 1 shows the general overveiw of the model.

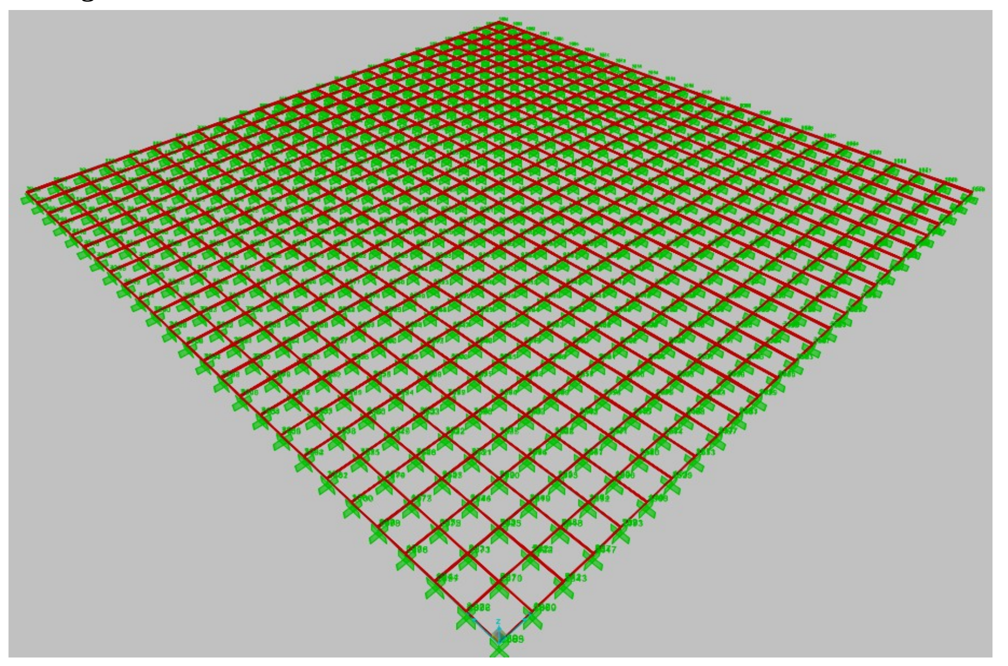


Figure (1): Finite element model for multi-layered flexible pavement.

Wheel loads are applied on both lanes directly on the asphaltic layer over a rectangular contact area with contact pressure of 345 kN/m^2 (50 psi). The model consists of non-linear link elements to form a virtual layer between the actual flexible pavement structural layers having properties the same as the underlying layer. In addition, the model consists of shell elements each having a dimension of 30 x 30 cm (12 x 12 inches) as shown in Figure 2. For the purposes of translating the forces and displacements from the asphaltic concrete layer to the underlying base layer, the virtual layer proposed above was represented by non-linear link elements.

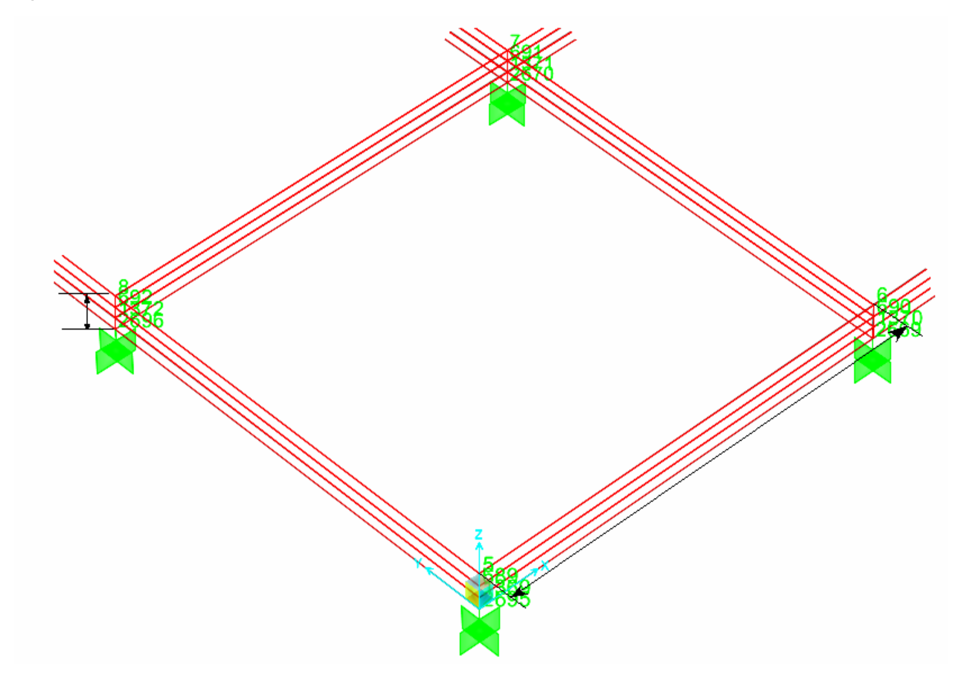


Figure (2): Dimentions of the elements considered in the model.

The base layer is represented by shell elements having an element size the same as the asphaltic concrete layer with different material properties. The subgrade layer is situated directly underneath the base layer represented by spring elements that have the same typical properties of subgrade stiffness. The boundary conditions used are best to

describe the function of the shoulder and the connection of the shoulder and flexible pavement structure in the longitudinal direction. The pine-ended (Hinge) restrain was proposed, which represents the existence of stabilized shoulders at both sides of the highway. This will provide confinement for the proposed model and prohibits lateral movements and, eventually reducing the overall stresses. The choice between free-ended and pine-ended (which both do not representing the true case), a pin-ended support is considered more realistic since it will prevent the translation movements in all directions, but allows for rotation to take place around one axis. The other two ends of the model are considered pine-ended because the effect of the wheel loads will disseminate at particular distance in which no observed settlement, stresses, or strains exists considering the fact that the assumed length of section is long enough for such observation.

5. Model Analysis

The criteria by which the structure is functioning based on the applied load are taken to be,

a) The lateral strain underneath the asphaltic concrete layer to represent the critical strain, it can be evaluated considering shell elements as follows:

$$\varepsilon_x = 1 / E \quad (\sigma_x - \upsilon\, \sigma_y) \qquad \text{Equation 1}$$

where:

ε_x = Strain underneath the AC layer

σ_x, σ_y = Stresses in the x and y direction

E = Young's modulus of elasticity

υ = Poisson's ratio

b) The vertical strain (in the vertical direction) at the top of the subgrade layer to represent the critical strain in the subgrade.

In the analysis of the model, some basic assumptions were considered (basically, assumptions considered in multilayer systems):

- Each layer is homogeneous, isotropic, and linearly elastic with an elastic modulus (E) and Poisson's ratio (**υ)**,
- The material of the layer is weightless and infinite in the x and y directions,
- Each layer has a finite thickness, but the lowest layer (subgrade) is infinite in thickness,
- A uniform pressure is applied on the surface, and
- Continuity conditions are satisfied at the layer interfaces of the same vertical stress, shear stress, vertical displacement, and radial displacement.

For the purpose of this research, properties of materials for surface and base are set at typical values of a modulus of elasticity of 4.14 x 10^6 kN/m^2 (600,000 psi) and 2.14 x 10^6 kN/m^2 (31,000 psi), respectively; and Poisson's ratio of 0.4 and variable values for the subgrade based on resilient modulus (Mr) (measure of the elastic property of soil recognizing certain non-linear characteristics) of natural roadbed soil. Subgrade spring constants vary based on the characteristics of natural soil. Values of resilient modulus ranged from 1.22 x 10^6 to 3.66 x 10^6 kN/m^3 (4,500 to 13,500 lb/in^3 respectively) to cover most practical soils encountered in the field as a subgrade. Due to the variability in the values of Mr, as a result of the varying moisture content through out the year, the methodology of effective roadbed soils resilient modulus considered by AASHTO were adopted (AASHTO 1993).

In the analysis of wheel loads, several combinations of load configuration were considered to arrive at a critical configuration of two axles lie in the same line in the lateral direction on the two lanes proposed. An equivalent 18-kips single axle load was adopted having a contact pressure of 345 kN/m^2 (50 psi) on equivalent rectangular area of 30 x 37.5 cm (approximately 12 x 15 inches). Considering the different types of axle configurations and loads, the model can adopt the approach

considered by AASHTO through the equivalent 18 kips single axle load (SAL) (load equivalency factors). Dynamic behavior of wheel loads were considered using the equivalent approach in which it is assumed that the response of a point at a distance x from a moving load is equal to the response of another point at the same distance x from the stationary load (Lin and Gazis 1999). Figure 3 shows the reduction in strain at the bottom of the AC layer as a function of vehicle speed. In general, an increase in speed of a vehicle decreases the tensile strain at the bottom of asphaltic concrete layer.

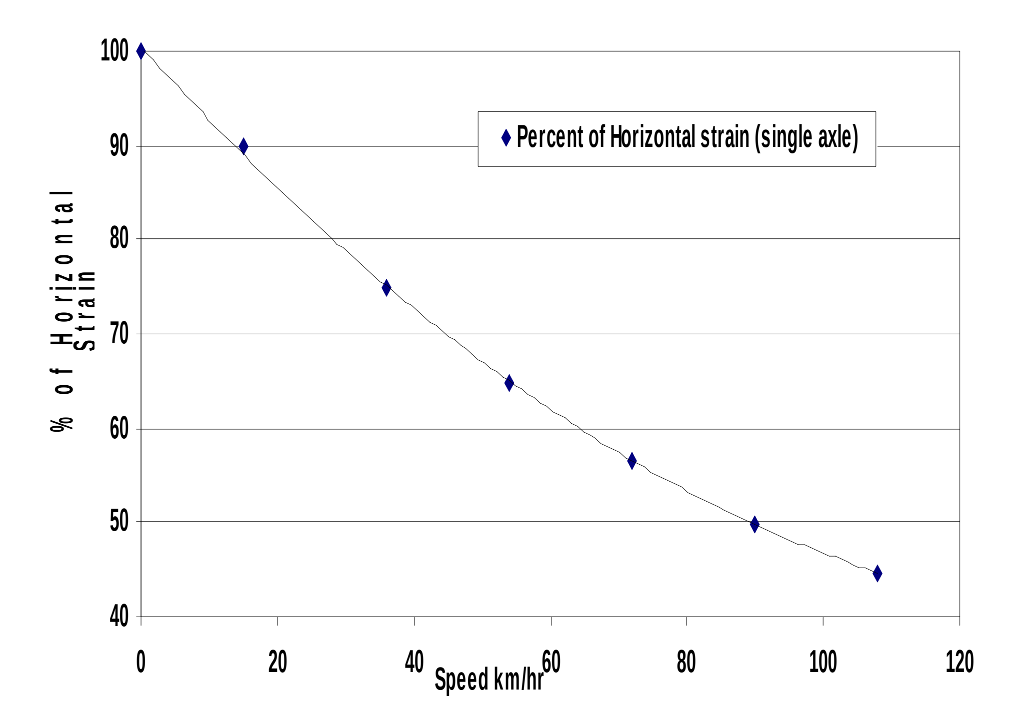


Figure (3): Percentage of horizontal strain under the asphaltic concrete layer for the various values of speed relative to static conditions (single axle).

6. Model Evaluation

The evaluation of the proposed model is based on trial-and-error iterations for several Equivalent Single Axle Loads (ESAL) repetitions for various values of Mr. Each set of iterations are based on certain assumptions of thickness to meet the limiting criteria. Elastic theory computations were used for the estimation of vertical subgrade strain criterion (Siddharthan 1998). This criterion relates the allowable vertical subgrade strain value for a given number of 18 kips SAL repetitions. The same theory was also used for developing of limiting strain criteria for fatigue cracking. The process is repeated to cover a wide range of axle loads as well as Mr values. Thickness of the three pavement layers are dealt with in terms of structural number (SN) as per the methodology adopted by AASHTO Guide for the Design of Flexible Pavement Structures using materials' coefficients and drainage coefficients to transform those thicknesses to SN values. The same process is repeated for the analysis of vehicle speed by applying the appropriate strain reduction based on vehicle speed. Several values of speed are considered to represent different classes of highways.

Based on the output of both considerations above, that is, static and dynamic analysis, a multiple regression analysis is performed with the structural number as the dependant variable and the velocity, Mr, and ESAL repetitions as independent variables. The multiple regression equation developed was calibrated using the AASHTO methodology for the case of static analysis since this methodology is mainly used for static loading in spite of its bases from the AASHTO Road Test.

Considering the outcome of the static case (velocity equals zero), the following multiple regression equation was developed:

$$SN = 0.513 \, Ln\,(ESAL) - 1.34 \log 10\,(Mr)_{sub} + 9.53 \qquad \text{Equation 2}$$

where:

SN = Structural Number.

ESAL = Equivalent Single Axle Load repetitions.

$(Mr)_{sub}$ = Resilient Modulus of subgrade material (lb/in^3). Figure 4 gives the results of the application of the finite element model relative to those of the AASHTO method for several cases of subgrade resilient modulus and single axle load repetitions. For the case of $M_r = 1.22 \times 10^6$ kN/m^3 (4,500 lb/in^3) for example, the results show a natural behavior of an increase in SN as the number of ESAL repetitions increases. In addition, Figure 4 shows this relation for values of resilient modulus of 2.0×10^6 , 2.9×10^6, and 3.7×10^6 kN/m^3 (7500, 10500, and 13500 lb/in^3, respectively). The curves reflect a realistic behavior in a drop in SN value as the Mr increases.

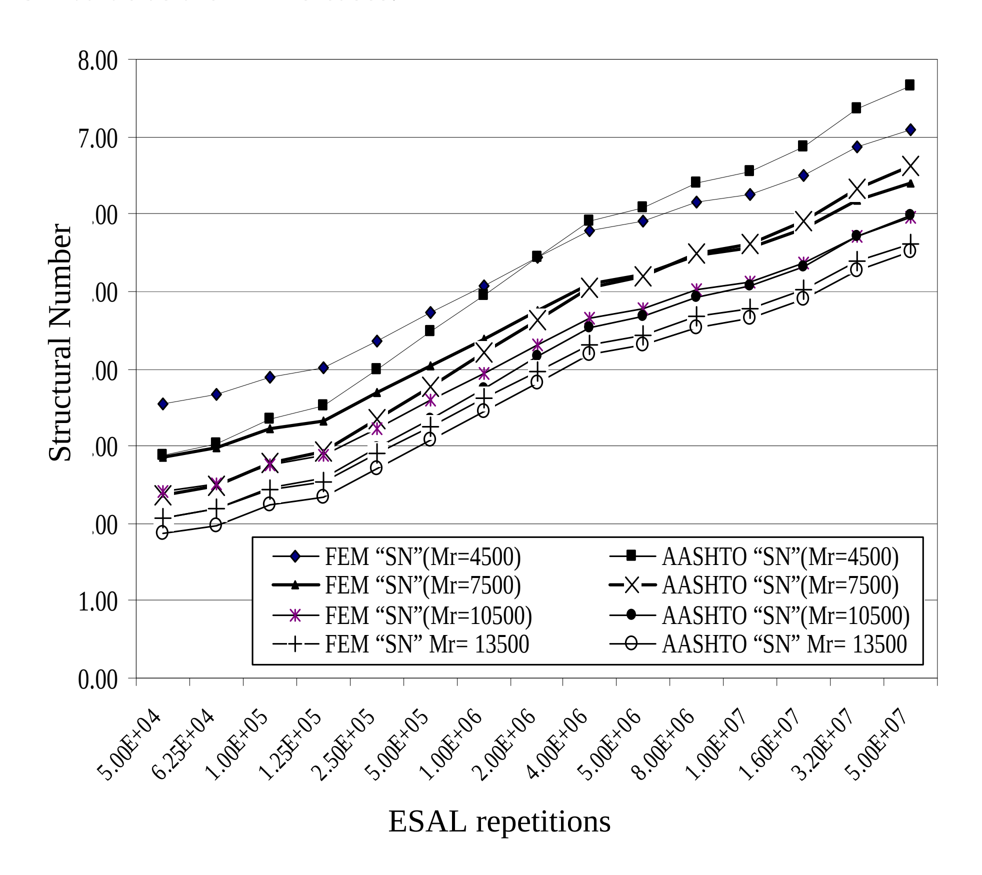


Figure (4): ESAL repetitions versus structural number for both design approaches (AASHTO & FEM).

The differences between the two methods can be analyzed through Figure 5. It is observed that a drop in the percentage difference exists as the single axle load repetitions increase considering the AASHTO results as a reference. This is due to the conservative results of the AASHTO method when incorporated for low truck traffic. In case of medium and high truck traffic (collectors and arterial highways) percentage differences narrow to a minimum followed by an increase up to eight percent for a resilient modulus of 1.22 x 10^6 kN/m^3 (4500 lb/in^3). The overall percentage difference between the two methods for all cases considered and for the ESAL above 1x10^6 repetitions ranges between -6 to approximately 7 % with an overall average of 1.3 %.

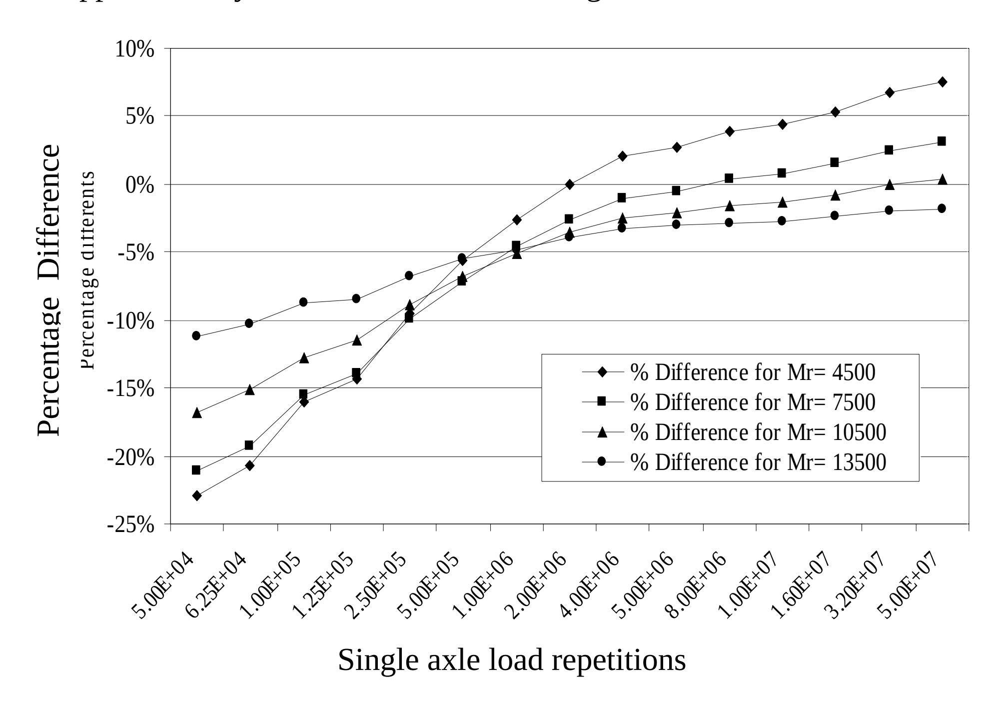


Figure (5): Axle repetitions versus percentage difference between the two methods used (AASHTO & FEM).

Based on the results of dynamic analysis considering the simulation of the moving wheel load using the equivalent approach as shown in Figure 2, the speed of the vehicle is integrated into the equation derived above. In order to arrive at a simplified approach that can be easily adopted by highway engineers, recommended speeds for the different classes of highways can be suggested. Minimum speeds suggested by AASHTO (AASHTO 1993) can be adopted as shown in Table 1. Speed studies can be performed in each urban or rural setting where appropriate decisions can be taken on a set of speeds most appropriate locally.

Table (1): Recommended minimum speed for different classes of highways.

Highway Class	**Recommended Speed (km/hr)**
Local	Up to 30
Collector	40-50
Arterial	50-60

Source: AASHTO Guide for Design of Pavement Structures, 1993.

Based on this methodology, the following equation was developed:

$SN = 0.44 \text{ Ln } ESAL - 1.74 \text{ Ln } (Mr)_{sub} - 0.14\, V^{1/2} + 14.71$ Equation 4

where:

SN = Structural Number

ESAL = Equivalent Single Axle Load repetitions

$(Mr)_{sub}$ = Resilient Modulus of subgrade material (lb/in^3)

V = Vehicle velocity (km/hr)

Figure 6 gives a comparative analysis of the design structural number for several values of design speeds relative to both design methods, mainly the proposed mechanical approach and the empirical AASHTO approach. Results indicate a reduction in the value of design structural number with an increase in design speed with a range of 11.0 to 5.4 %

per unit of speed considered. For example, for a major arterial, the reduction in design structural number can reach as much as 22 %. On the other hand, for local roads, a reduction in design structural number can reach 11% of that of the static case. For comparative analysis, those values can reach approximately 6 and 33 % reduction in design structural number compared to AASHTO design method for major arterials and local roads, respectively.

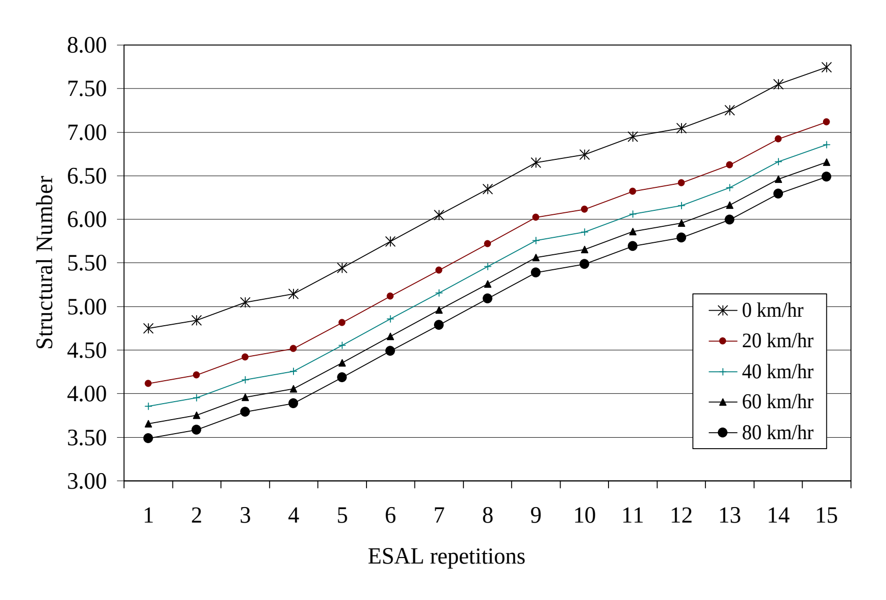


Figure (6): Structural Number versus ESAL repetitions for values of vehicle speeds (Mr = 4500 lb/in^3).

Generally, the results achieved in the static as well as that of the dynamic analysis agree with studies that considered the effect of vehicle speed and field axle loads on the measured tensile strains (Timm et al. 1998, Roberts 1989, Sven et al. 1992, Zafir et al. 1994, Lin and Gazis 1999, Dorman 1965). In addition, the outcome of this research gives a practical method of designing flexible pavement structure, yet it depends partly on the mechanical approach which opens the way for the utilizing the changing variables introduced in the analysis of pavement structures.

7. Conclusions and Recommendations

Based on the results obtained in this research, the followings are concluded:

1. The suggested finite element model in this research gives comparable results relative to the AASHTO method for the design of pavement structures.
2. The derived simplified M-E approach utilizing both finite elements and the AASHTO empirical procedure is considered an effective and practical tool for the design of highway structures.
3. Incorporating speed of axle loads into the derived M-E approach can encourage better utilization of other changing parameters into the design of flexible pavement structures and provide a better utilization of materials in roadway construction.
4. Further research should consider the incorporation of other variables into the design using the finite element approach such as equivalent axle wheel load repetitions, variation in the moisture content of the subgrade, and vehicular speeds.
5. Interactive software can be developed using the basic input variables used in pavement design with a capability of analysing traffic data, material properties, and local geometric conditions based on the finite element models.

List of Notation

Notation	Definition
km/hr	Kilometer per hour
kN/m^3	Kilo-Newton per cubic meter
mm	Millimeters
lb/in^3	Pounds per cubic inch
mph	Miles per hour
kN/m^2	Kilo-Newton per square meter
psi	Pounds per square inch

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